APPENDIX

The series to be summed is

$$\sum_{n=1,3\cdots}^{\infty}\left(1+\coth n \ln \frac{b}{a}\right)\frac{\sin^2 n\alpha}{n^3}.$$

This series may be written as:

$$2\sum_{1,3\cdots}^{\infty}\frac{\sin^2 n\alpha}{n^3}+\sum_{1,3\cdots}^{\infty}\left(\coth n \ln \frac{b}{a}-1\right)\frac{\sin^2 n\alpha}{n^3}$$

where the second series is rapidly converging and in the form of a correction term. Integrating the well known geometric series

$$\sum_{1}^{\infty} e^{2jn\alpha}=\frac{e^{2j\alpha}}{1-e^{2j\alpha}}$$

once with respect to α and taking the real part gives

$$\sum_{1}^{\infty}\frac{\cos 2n\alpha}{n}=-\ln 2\sin\alpha.$$

Replacing 2α by $\pi-2\alpha$ and adding and subtracting series, it is readily deduced that

$$2\sum_{1,3\cdots}^{\infty}\frac{\cos 2n\alpha}{n}=-\ln\tan\alpha.$$

Integrating this series twice with respect to α gives

$$\sum_{1,3}^{\infty}\frac{\sin^2 n\alpha}{n^3}=\int_0^{\alpha}\int_0^{r}\ln\tan y\,dy\,dx.$$

For a limited range of α, $\ln\tan y$ may be replaced by the first few terms of its Maclaurin's expansion and the integration may then be performed. One has

$$\ln\tan y=\ln y+\frac{y^2}{3}+\frac{7y^4}{90}+\cdots.$$

The integration gives

$$\sum_{1,3\cdots}^{\infty}\frac{\sin^2 n\alpha}{n^3}=(1.5-\ln\alpha)\frac{\alpha^2}{2}-\frac{\alpha^4}{36}-\cdots.$$

For $\alpha \leqq .5$ the following result is obtained:

$$\sum_{1,3\cdots}^{\infty}\left(1+\coth n\ln\frac{b}{a}\right)\frac{\sin^2 n\alpha}{n^3}$$
$$=(1.5-\ln\alpha)\alpha^2+\sum_{1,3}^{\infty}\left(\coth n\ln\frac{b}{a}-1\right)\frac{\sin^2 n\alpha}{n^3}.$$

Broadband Ferrite Microwave Isolator*

P. H. VARTANIAN†, J. L. MELCHOR†, AND W. P. AYRES†

Summary—**A new type broadband unidirectional transmission line has been built utilizing the difference in energy distribution between two counter-rotating circularly polarized waves in a circular waveguide containing a ferrite. This principle of isolation is different from those which have been used previously.**

A large difference is observed in the energy distribution of two counter-rotating TE_{11} modes in a ferrite loaded circular waveguide. A ferrite rod magnetized along its axis presents an effective rf permeability of approximately two for the mode rotating in a negative screw sense with respect to the direction of magnetization. For the positive sense of rotation the effective rf permeability becomes very small and negligible energy is transmitted through the ferrite rod.

Unidirectional transmission characteristics were achieved by adding quarter wave plates before and after the ferrite rod and inserting an absorber into the ferrite. For the direction of propagation for which the quarter wave plate converts from a linear input to a positive circular rotation the positive wave tends to go around the ferrite with small loss. For the other direction of propagation the quarter wave plate converts the linear input wave to a negative wave which tends to concentrate in the ferrite and is absorbed.

Based on the principles described, an isolator was constructed which gives better than 30 db isolation over the range 8 to 11 kmc. The insertion loss is less than 2 db from 8 to 10.5 kmc and increases to 3 db at 11 kmc. The complete unit is 10½ inches long and weighs 2¼ pounds.

The main advantage of this isolator over present transverse field rectangular waveguide isolators and Faraday rotation isolators is its improved bandwidth. Other advantages are that the isolator is not sensitive to changes in magnetic field and it operates with a readily obtainable ferrite at low magnetic fields. Its vswr over the band is less than 1.2. The principle of this isolator is applicable to other frequency bands.

INTRODUCTION

TO KEEP abreast of current systems developments, both manufacturers and users of microwave components have felt the need for broadband microwave isolators. In attempts to make practical microwave isolators, ferrites have been heavily exploited in both circular and rectangular waveguide geometries. Effects of differential phase shift and differential resonance absorption for two directions of wave propagation are widely used as the basis for isolation in both waveguide geometries. In most reported cases, however, the bandwidth of these devices does not exceed ten per cent.

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A new approach to microwave isolation in circular waveguide is reported here which does not not employ either Faraday rotation or magnetic resonance absorption. Instead, energy distribution differences for the two counter-rotating TE_{11} modes in a ferrite loaded circular waveguide are utilized as a basis for isolation. Energy distribution in a rectangular waveguide loaded with a transversely magnetized ferrite slab has been discussed by Lax *et al*;[1] but due to its added complexity the problem has not been solved in circular waveguide. Consequently the boundary value solution will not be discussed, but analogy will be made with waves propagating in infinite magnetic media and with waves propagating along an isotropic dielectric waveguide. With these analogies an attempt is made to give the reader a qualitative feeling for the energy distribution in ferrite loaded circular waveguide and to explain its applications in microwave isolators and other devices.

THEORY OF OPERATION

Experimental evidence has been found by the authors[2] in agreement with the Fox and Weiss[3] theory that the microwave energy distribution in a waveguide containing a magnetized ferrite is different for the two senses of circular polarization. The reason for this can be discussed qualitatively by examining the effective permeability seen by each of the rotating waves. Rotating waves are the normal modes of the problem in that the permeability is a scalar only for the case of circular polarization. For circularly polarized plane waves in a lossless infinite ferrite medium the scalar permeabilities are given by

$$\mu_\pm = 1 + \frac{\gamma 4\pi M_s}{\gamma H \pm \omega} \tag{1}$$

where $4\pi M_s$ is the saturation magnetization, H is the applied magnetic field, γ is the gyromagnetic ratio, a negative number, and ω is the angular frequency. The symbol μ_+ denotes effective permeability for a wave rotating in the right hand direction looking along the magnetic field and μ_- denotes permeability for the counter-rotating wave.

Fig. 1 shows the two permeabilities of (1) as a function of magnetic field. It is seen that over a broad range of frequencies and magnetic fields, the negative polarization permeability is essentially two while the positive polarization permeability is very small. The μ_+ may actually in certain regions become zero or negative. For these regions it is necessary to include also the imaginary part of the permeability and the propagation constant is now more complicated but never less than zero.

Since the dielectric constant of most ferrites is in the range of 10–15 the negative circular polarization sees a large $\mu\epsilon$ product and the ferrite rod acts essentially as a dielectric waveguide. Consequently, if a ferrite rod is placed in a circular or square waveguide, the energy of a negatively rotating wave will be concentrated in or near the rod. On the other hand, the positive circular polarization sees a small $\mu\epsilon$ in the region of the ferrite and hence the energy passes mainly around the rod.

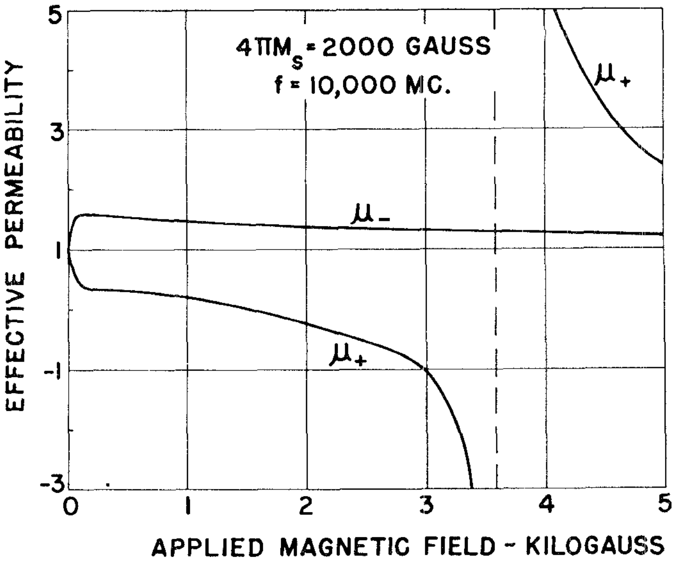

Fig. 1—Effective permeabilities for positive and negative circular waves in infinite lossless medium as a function of applied magnetic field.

There are several ways in which one of the two circular components may be selectively absorbed. A lossy material may be placed around the ferrite to fill the waveguide and absorb the positive wave; or an absorber may be introduced on the surface or inside the ferrite to absorb the negative wave.

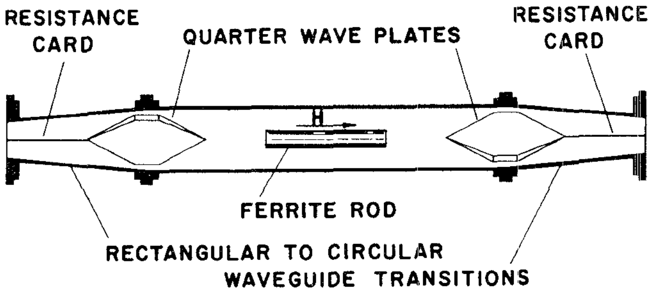

Fig. 2—Waveguide geometry used for loss measurements and used in construction of isolator.

DESCRIPTION OF ISOLATOR

An isolator was made by placing an Aquadag coated ferrite rod of the optimum diameter for the frequency band of interest between two quarter wave plates in a cylindrical waveguide as shown in Fig. 2. Ferrite, General Ceramics *R*-1, is magnetized to a low value with a longitudinal field produced by a permanent magnet.

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[1] B. Lax, K. J. Button, and L. M. Roth, "Ferrite phase shifters in rectangular waveguide," *J. Appl. Phys.*, vol. 26, pp. 1413–1421; November, 1954.

[2] J. L. Melchor, W. P. Ayres, and P. H. Vartanian, "Energy concentration effects in ferrite loaded waveguides," *J. Appl. Phys.*, vol. 27; January, 1956.

[3] A. G. Fox and M. T. Weiss, "Discussion on ferromagnetic Faraday effect," *Rev. Mod. Phys.*, vol. 25, pp. 262–263; Janaury, 1955.

The energy from the generator being isolated is converted by the first quarter wave plate into positive circular polarization which is transmitted essentially unattenuated and reconverted to linear polarization by the second quarter wave plate. Reflected energy from the load on the other hand is converted into negative circular polarization relative to the applied magnetic field and is attenuated in the absorber on the ferrite surface. Alternately the absorber may be put outside the ferrite to interact with the negative polarization. This has not performed as well since it requires a good match into the ferrite rod. The matches for the case of the absorber in the ferrite are not as important. This is because the discontinuity for the forward wave is very small since the permeability is small. The coated ferrite rod looks simply like an absorbing needle in the center of the guide which does not interact with the TE_{11} mode. For the backward wave there is a large discontinuity however, and reflections are absorbed in an *H* plane resistance card on the load side of the quarter wave plate. The quarter wave plate converts energy reflected from the ferrite into the *H* plane.

The quarter wave plates are of tapered polystyrene and are designed to give wide bandwidth. They have an ellipticity (ratio of space quadrature fields) of less than 2 db over the 8–11 kmc band. It can be shown that the forward loss caused by the quarter wave plate not converting all the energy into the proper polarization is

$$\frac{P_{\text{loss}}}{P_{\text{inc}}} = 1 - \frac{1}{4} \frac{(e+1)^4}{(e^2+1)^2}$$

where e is the ellipticity voltage ratio. Thus a 2 db ellipticity causes a loss of only 0.11 db in the forward direction for the two quarter wave plates.

The rectangular to circular waveguide transitions are approximately $2\frac{1}{4}$ inches long and have a vswr of less than 1.2 from 8 to 11 kmc. Tapered resistance cards are placed in the *H* plane of the transitions.

Isolation Characteristics

There are many modifications possible in the device shown in Fig. 2. It was found that the characteristics are sensitive to absorber position, rod diameter, absorber resistance, magnetic field, and ferrite used.

Absorber Position

As stated above, there are three possible places to locate the absorber: in the ferrite, on the ferrite, and outside of the ferrite. The absorber was placed inside of the ferrite by slicing a $\frac{1}{4}$-inch ferrite rod longitudinally and coating an absorbing material on the interfaces. The isolation is seen in Fig. 3 to be better than 15 db from 8 to 11 kmc with an insertion loss of less than 1.5 db. From 9 to 10 kmc the isolation exceeds 25 db while the insertion loss is less than 1 db. These data were taken with a solenoid adjusted for 375 gauss.

Isolators constructed with the absorbing material located between the ferrite and the waveguide wall were found to be unsatisfactory. Suitable isolation was achieved with this configuration but the insertion loss was found to be much too large. This can be attributed to losses due to a poor match between the waveguide and the ferrite rod which must act as a dielectric waveguide for the forward wave. These poor matches can also set up cavity resonances within the ferrite. Isolators constructed with the absorber outside the ferrite were compared extensively with those constructed with the absorber inside the ferrite. Rods with diameters as large as $\frac{1}{2}$ inch were used with the absorber outside. For all diameters between $\frac{1}{8}$ inch and $\frac{1}{2}$ inch the insertion loss was high except at discrete frequencies for larger diameter rods where the ferrite acted as a transmission cavity.[2]

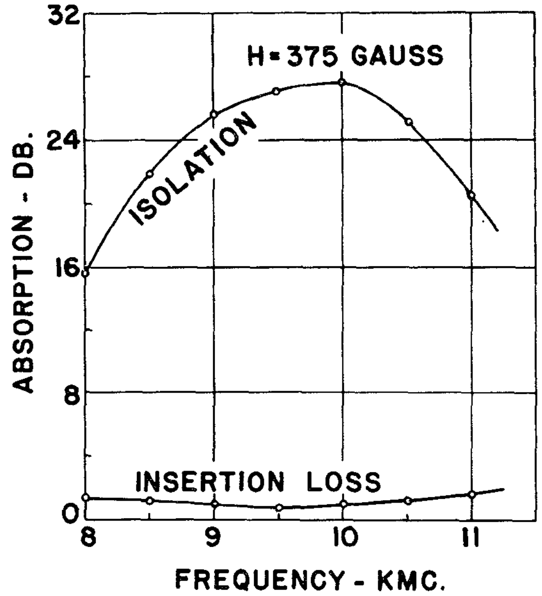

Fig. 3—Isolation and insertion loss versus frequency for isolator constructed with 0.250-inch diameter ferrite rod 2 inches long. Rod sawed in half longitudinally and absorber applied to interfaces.

The simplest and most convenient form of applying the absorber was to coat Aquadag on the ferrite surface. With this absorber configuration the insertion loss is still fairly low and the rod shape is preserved. The saw cut required with the absorber inside distorts the rod shape and decreases its effective diameter. As will be shown later in a discussion of the effects of diameter this can be important. In the remaining data to be discussed the absorber was coated on the ferrite surface.

Ferrite Rod Diameter Effects

As pointed out in the Theory of Operation, the ferrite rod begins to act as a dielectric waveguide for the negative rotating wave. Consequently we should expect operation of the isolator to be sensitive to rod diameter.

Curve *A* in Fig. 4 illustrates a 0.125-inch diameter rod beginning to act as a dielectric waveguide as frequency increases. The rod is 4 inches long, magnetized longitudinally, and coated with an absorber. The absorption of the negative wave increases with increasing frequency indicating that as frequency increases the ferrite begins to act more and more as a dielectric waveguide. This diameter rod would be useful in an isolator operating above 12 kmc but operation below 12 kmc requires larger diameter ferrite rods.

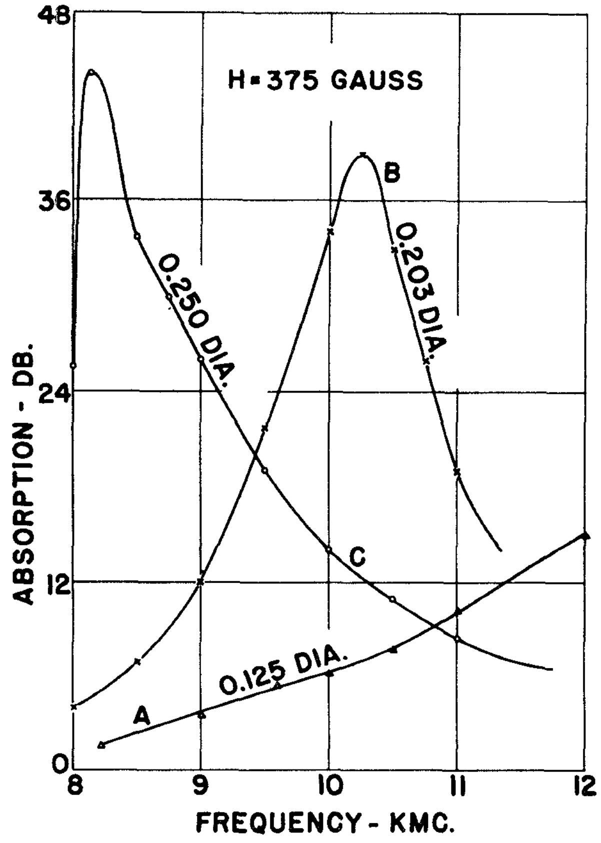

Fig. 4—Absorption losses versus frequency of negative wave for three different diameter rods with absorber coated on surface.

Curves *B* and *C* for rods of 0.203- and 0.250-inch diameters exhibit peaks in *X* band as shown in Fig. 4. Thus there is an optimum band of frequencies for dielectric waveguide effects for each ferrite diameter. The frequencies at which maximum isolations occur for a series of different diameter coated rods are shown in Fig. 5. Here it is seen that as diameter increases the frequency of the peak decreases. Fig. 5 indicates that rod diameters ranging from 0.155 to 0.250 inch will give isolation peaks distributed across the 8.2 to 12.4 kmc band. In practice three different diameter rods, each two inches long, are required to give better than 25 db isolation across this frequency band. However, with this arrangement the forward or insertion loss increases rapidly at the high frequency end of the band.

Surface Resistivity Effects

The isolation characteristics were found to depend strongly on the ferrite surface coating resistivity. A low surface resistivity resulted in less isolation, higher insertion loss, and a broader isolation curve as a function of frequency. Increased resistivity yielded lower insertion loss and a sharper isolation curve which reduced to isolation peaks due to reflection from the ferrite rod for very thin absorber films. An optimum resistance of roughly 2,000 ohms was found when measured from one end to the other on the periphery of a $\frac{1}{4}$-inch diameter rod, 2 inches long.

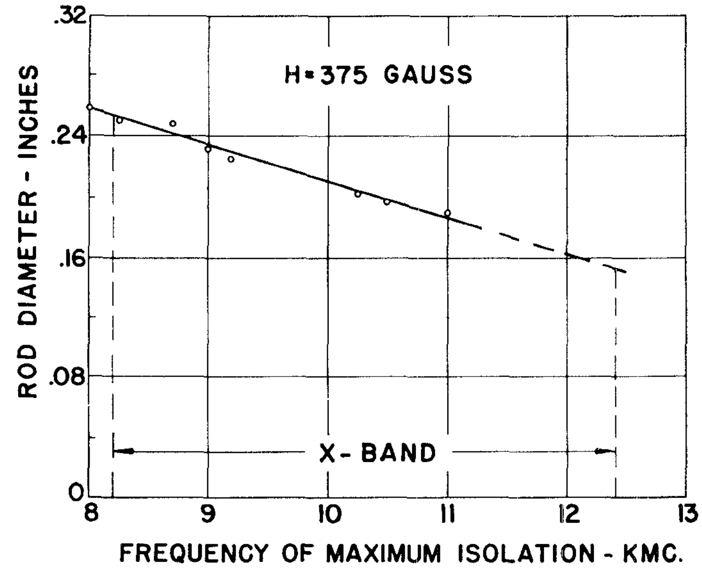

Fig. 5—Frequency of maximum isolation as a function of diameter of ferrite rods with absorber coated on the surface. Waveguide diameter is 0.93 inch.

Typical Isolator and Characteristics

An isolator was constructed with two ferrite rods of 0.250- and 0.190-inch diameters and 2 inches long placed end to end. Characteristics of this isolator are shown in Fig. 6. An isolation of greater than 30 db was achieved from 8 to 11 kmc with an insertion loss of less than 2 db from 8 to 10.5 kmc and increasing to 3 db at 11 kmc. The unit consists of two rectangular to circular waveguide transitions $2\frac{1}{4}$ inches long, two quarter wave plates, 6 ring magnets of the type Indiana Steel R-142, two pieces of resistance card, a circular waveguide section 6 inches long, and two absorber coated ferrite rods of 0.250- and 0.190-inch diameters. A photograph of the unit is shown in Fig. 7. It weighs $2\frac{1}{4}$ pounds and has an over-all length of $10\frac{1}{2}$ inches.

Merits of Isolator

An isolator based on differences in energy distribution for the rotating waves appears to be more broadband than a Faraday rotation isolator in the same geometry.

In addition the material requirements for this type of isolator are less stringent. Since the transmitted wave tends to pass around the ferrite there are less dielectric losses than for a Faraday rotator where half the energy is in the opposite sense of circular polarization and goes into the rod. In fact a ferrite with higher dielectric losses would be useful for this application.

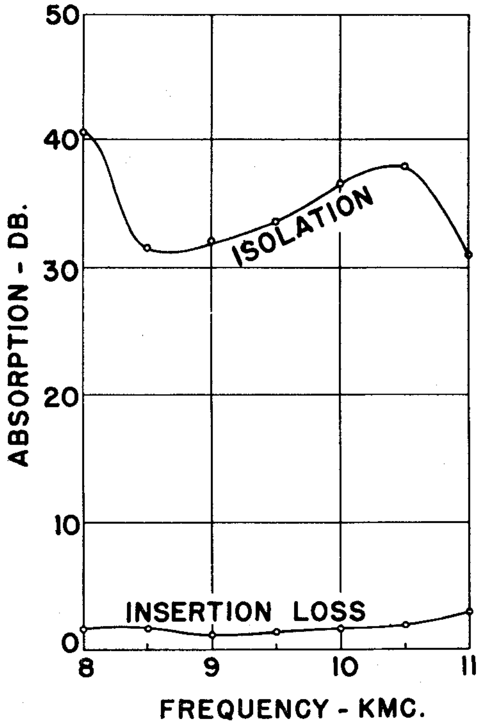

Fig. 6—Absorption characteristics of isolator constructed with 0.250-inch and 0.190-inch diameter rods.

The isolator is quite insensitive to magnetic field as expected from the effective permeability curves of Fig. 1. This makes it more practical for use with permanent magnets for which the magnetization changes with time. The magnetic field required is much smaller than that required with the resonance type isolator where fields of 2,000 to 3,000 gauss are commonly employed.

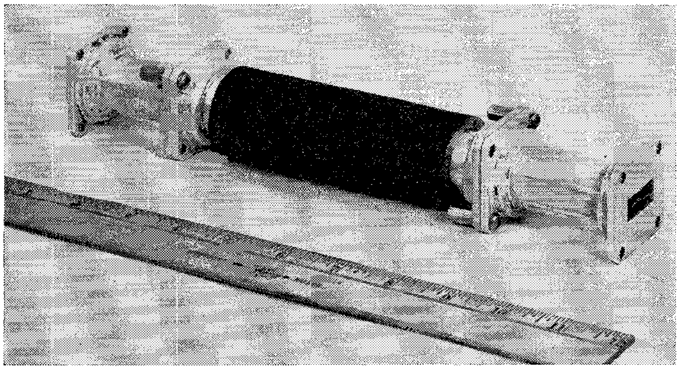

Fig. 7—Photograph of isolator.

Since the loss occurs at the ferrite surface the problem of heat dissipation is less severe than for the other cases. With a metalized resistive coating and forced air cooling the potential high power applications are promising. A ferrite is a poor thermal conductor, making it difficult to extract energy absorbed within it by electric or magnetic losses. This type of isolator tends to reduce the losses which occur within the ferrite and reduces the problem of heat dissipation.

The isolator, which weighs 2¼ pounds, is considerably lighter than resonance absorption isolators, but comparable to Faraday rotation isolators. It has an advantage over the Faraday rotator in that any desired isolation, or reverse attenuation, can be achieved simply by making the ferrite element longer. Isolation was found to be directly proportional to the length of the element.

Limitations of Isolator

The principal limitation is the increase in insertion loss at higher frequencies. As seen from Figs. 3 or 6 the insertion loss begins to increase rather rapidly between 10 and 11 kmc. Upon examining individual isolation and insertion loss curves for different coated rod diameters it was found that the frequency for peak isolation is roughly 2 kmc below the frequency for which the sharp increase in insertion loss begins. In Fig. 6 the insertion loss at 11 kmc is due primarily to the ¼-inch rod. It increases continuously through 12.4 kmc which was the upper limit of the measurements. The increase is attributed to the fact that the effective permeability for the positive wave does not go to a sufficiently low value to exclude that wave from the ferrite at higher frequencies. To substantiate this hypothesis the applied magnetic field was increased in several steps to 1,500 gauss. At each successively higher field value the increased insertion loss occurred at a higher frequency. This indicates a reduction of the effective permeability for the positive wave which makes the rod's electrical size smaller, requiring higher frequencies for dielectric waveguide effects.

To avoid the problem of increasing insertion loss a material with lower μ_+ at magnetic saturation is desired. The field value for saturation is chosen because μ_- is a maximum there and it is an easily achievable magnetic field. At that point (1) becomes

$$\mu_{\pm} = \frac{\gamma B_s \pm \omega}{\gamma H_s \pm \omega} \tag{2}$$

where H_s is the magnetic field required to saturate the ferrite and B_s is the saturation flux density, $H_s + 4\pi M_s$. From this expression it is seen that μ_+ approaches zero as $|\gamma| B_s$ approaches ω. For X band operation a saturation moment greater than 3,000 gauss will give a more favorable ratio of propagation constants. Ferramic $R-1$ has a saturation moment of only 2,000 gauss.

CONCLUSION

Using the difference in energy distribution for circular waves transmitted through ferrite loaded circular waveguide, an isolator was constructed with 30 db isolation from 8 to 11 kmc. Its insertion loss is less than 3 db and it does not appear to be unduly critical with respect to any of the operating parameters. By providing means for varying the applied field the isolator becomes an amplitude modulator or electronic switch.

Since the energy transmitted in the forward direction tends to go around the ferrite, low magnetic and dielectric losses occur in it. This approach to isolation is promising for higher powers. A more suitable choice of material will reduce the insertion loss presently observed.

The frequency of maximum isolation for a single ferrite rod is inversely proportional to the ferrite diameter. Various diameter rods can be added in series to increase the isolation bandwidth. However, with the ferrite used an increase in insertion loss on the high frequency side discourages this beyond a 3 kmc bandwidth. The increase in insertion loss which occurs at higher frequencies is attributable to dielectric waveguide effects for the positive wave. With a dielectric constant of 13 the $\mu\epsilon$ product is large even if μ is as low as 0.3. In such a case, dielectric waveguide type transmission is expected to become noticeable at 11 kmc.

The use of differential energy distribution for nonreciprocal and magnetically controllable circuit elements is promising from the standpoint of stability, bandwidth, and power handling capacity.

ACKNOWLEDGMENT

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An Approximate Analysis of Coaxial Line with a Helical Dielectric Support*

J. W. E. GRIEMSMANN†

SHOWN in Fig. 1 is a cutaway section of coaxial cable with a helical dielectric support. The particular cable depicted bears the trade name of Styroflex[1] derived from the fact that the dielectric helix is built up in a winding operation from nonplasticized polystyrene tapes giving to the final assembly a tight grip on the center conductor and a good degree of allowable bending capability. The outer conductor in the original design of the cable consists of an aluminum sheath extruded over the dielectric. Other forms of cable with helical dielectric support are also available.

This type of cable is of interest as an alternative to broadband bead supported line, particularly for applications requiring long lengths of line or small diameter cable where multiplicity of bead supports can lead to high wave reflection characteristics in frequency bands of interest.

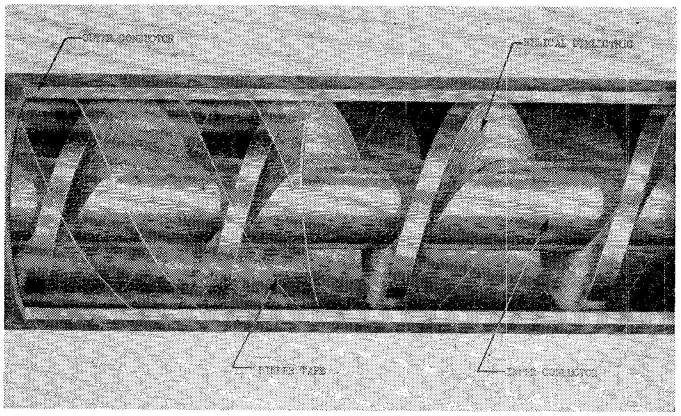

Fig. 1—Cut-away section of Styroflex cable.

The analysis given below shows that the total propagation in the helical line can be considered to be made up of two component propagations, one following the dielectric helix down the transmission line and the other following a helical path perpendicular to the dielectric. The latter type of propagation is that of an iterative transmission line and introduces for the overall propa-

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[1] Phelps Dodge Copper Products Corp.